Department of Mathematics

MAL 111 (2012 September) Minor Test 1

Time: 1 hour **Maximum Marks: 22**

Instruction: 1. Every question is compulsory.
2. No marks will be awarded unless necessary arguments are provided.

(1) Suppose for each $n \in \mathbb{N}$, the subset A_n of $\mathbb{C}$ is countable. Let $A = \bigcup_{i=1}^{\infty} A_i$. Recall (do not prove): "There is a surjective map $\psi : \mathbb{N} \to \mathbb{N} \times \mathbb{N}$."
Use it to prove that A is countable. [3]

(2) If A is a (Dedekind) cut containing 0 then define

$$-A := \{x - y : -x \in \mathbb{Q} \setminus A, y \in \mathbb{Q}, y > 0\}.$$

Let $P_r = \{x \in \mathbb{Q} : x < r\}$ for $r \in \mathbb{Q}$. Show that P_r is a cut and $-P_r = P_{-r}$ if $r > 0$. [2]

(3) Show that the intersection of any two open sets in a metric space is an open set. Produce an example to show that the intersection of infinitely many open sets is not necessarily open. [3]

(4) Suppose V is the open interval $(0, 1)$. For each $n \in \mathbb{N}$, define the subset T_n of $\mathbb{R}^2$ by $T_n = \{\frac{1}{n}\} \times V$. Let $T = \bigcup_{n=1}^{\infty} T_n$. Determine (with appropriate arguments) T', $\overline{T}$, Int$T$, Ext$T$ and BdT as a subset of the metric space $\mathbb{R}^2$ with usual metric. [5]

(5) Let $X = \mathbb{R}$ be the metric space with the usual metric on $\mathbb{R}$. Let $Y = \mathbb{R}$ be the metric space with discrete metric on $\mathbb{R}$. Define the maps $f : X \to Y$ and $g : Y \to X$ by $f(x) = x \ \forall\, x \in X$ and $g(x) = x \ \forall\, x \in Y$. Determine (with appropriate arguments) whether f and g are continuous. [2]

(6) Define the metric d on $X = \{1/n : n \in \mathbb{N}\}$ by $d(a, b) = |a - b|$. Determine what are all open sets and closed sets in X. [3]

(7) Show by using Nested Intervals Theorem that the closed interval $[0, 1]$ is uncountable. [4]

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Int $A = \phi$